

# Characterizing correlations with full counting statistics: classical Ising and quantum XY spin chains

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We propose to describe correlations in classical and quantum systems in terms of full counting statistics of a suitably chosen discrete observable. The method is illustrated with two exactly solvable examples: the classical one-dimensional Ising model and the quantum spin-1/2 XY chain. For the one-dimensional Ising model, our method results in a phase diagram with two phases distinguishable by the long-distance behavior of the Jordan–Wigner strings. For the quantum XY chain, the method reproduces the previously known phase diagram.

**1. Introduction.**— Thermodynamic phases are traditionally described by correlations of local observables. In the simplest case of phase transitions associated with symmetry breaking, phases are distinguished by the expectation value of a local order parameter. In more subtle situations (e.g., Kosterlitz–Thouless phase transition), it is the decay of correlations at large distances that distinguishes between the phases. In recent years, it was realized that other, more sophisticated characteristics of correlations may be useful: e.g., the notion of “topological order” (involving nonlocal order parameters) or entanglement entropy (in the case of quantum systems).

In the present work, we consider yet another *non-local* characteristics of correlations based on the full-counting-statistics (FCS) approach [1] (a related problem of order-parameter statistics was studied in Ref. 2). It was pointed out recently (in the context of temporal correlations) that analytical properties of the *extensive part* of FCS may be used to distinguish between different thermodynamic phases [3]. Here we apply this idea to spatial correlations and illustrate it with two examples: the classical Ising and the quantum XY spin chains (see also Ref. 4 for an example of one-dimensional free fermions, which do not exhibit any phase transition).

**2. FCS characterization of thermodynamic phases.**— In our approach, a thermodynamic phase is characterized by the singularities of the extensive part of a suitably defined FCS generating function  $\chi_0(\lambda)$ . This construction is applicable to any infinite system (either classical or quantum, not necessarily one-dimensional) which is periodic in space and possesses an extensive observable taking *quantized discrete values* (e.g., the number of particles or the projection of total spin on a given axis). Consider a large subsystem  $\Sigma$  containing  $N$  unit cells of the infinite system. Let  $Q$  be our discrete observable restricted to this subsystem and normalized to take integer values. Then one can construct the FCS generating function for the observable  $Q$ ,

$$\chi_\Sigma(\lambda) = \sum_m P_m e^{i\lambda m}, \quad (1)$$

where the sum is taken over all integer numbers  $m$  and  $P_m$  is the probability for the observable  $Q$  to take the value  $m$ . The generating function  $\chi_\Sigma(\lambda)$  has the form of a partition function [3] and therefore must depend exponentially on the size of the system  $\Sigma$  [5]:

$$\chi_\Sigma(\lambda) \propto \chi_0(\lambda)^N. \quad (2)$$

Here  $\chi_0(\lambda)$  plays the role of the *extensive part* of  $\chi_\Sigma(\lambda)$ . It is periodic in  $\lambda$  (with period  $2\pi$ ), but does not have to be smooth, and it is the singularities of  $\chi_0(\lambda)$  at real values of  $\lambda$  that we propose to use as a characteristics of the thermodynamic phase [3].

Note that the definition above is quite general and applies to a vast number of statistical and quantum problems (in many situations, one even has a choice between different possible observables  $Q$ ). In Fig. 1, we show two such examples: particles on a lattice (with  $Q$  being the number of particles) and a spin-1/2 chain (with  $Q$  being the number of up spins).

Our FCS construction is related to other characteristics of correlations. The analytic continuation  $\lambda \rightarrow i\infty$  produces the “emptiness formation probability” (EFP) [6, 7]. On the other hand, in quantum systems of non-interacting fermions, FCS is known to be related to the entanglement [8].

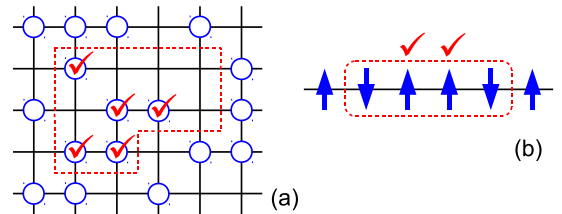


FIG. 1: Two examples of FCS in statistical or quantum systems. In both examples, the subsystem  $\Sigma$  is encircled by a dashed line. **(a)** Particles on a lattice,  $Q$  is the number of particles (in the configuration shown,  $Q = 5$ ). **(b)** Spin-1/2 chain,  $Q$  is the number of up spins (in the configuration shown,  $Q = 2$ ).

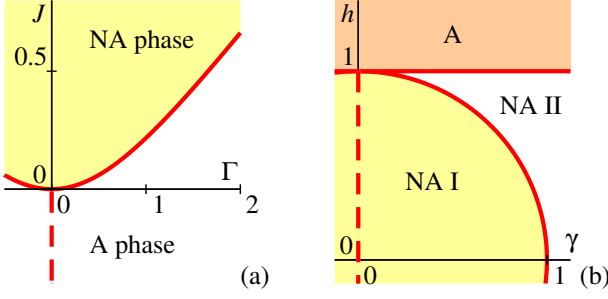


FIG. 2: (a) The FCS phase diagram of the classical Ising chain. The shaded upper region is the “nonanalytic” (NA) phase, and the lower region is the “analytic” (A) phase [10]. The phase diagram is symmetric with respect to the change of sign of  $\Gamma$ . (b) The FCS phase diagram of the spin-1/2 XY chain at zero temperature. The three phases are: Nonanalytic I, Nonanalytic II, and Analytic. On the dashed line  $\gamma = 0$ ,  $|h| < 1$ , the dependence of FCS on  $\gamma$  is nonanalytic. The phase diagram is symmetric with respect to the (independent) changes of sign of  $\gamma$  and  $h$ .

Singularities in  $\chi_0(\lambda)$  is a subtle characteristics of FCS. In Ref. 3 we argued that they are related to pre-exponential factors in staggered cumulants of the observable  $Q$  (if the singularity occurs at  $\lambda = \pi$ ). In one-dimensional systems, we can interpret  $\ln \chi_0(\lambda)$  as the inverse correlation length of the Jordan–Wigner string  $\exp(i\lambda Q)$  (such correlation functions were also discussed in the context of integrable systems [6]). In some situations, Jordan–Wigner strings may be related to physical quantities which are directly observable, e.g., spin correlations in the example of the XY chain below.

We propose to classify thermodynamic phases by the number and type of singularities of  $\chi_0(\lambda)$  at real values of  $\lambda$ . Thus obtained *FCS phases* do not have to exactly reproduce the conventional phase diagram. Below, we illustrate this proposal with two examples.

3. *One-dimensional Ising model.*— Our classification results in a nontrivial phase diagram for the one-dimensional Ising model. We consider the classical Ising chain in an external field described by the Hamiltonian

$$H = J \sum_j \sigma_j \sigma_{j+1} + \Gamma \sum_j \sigma_j, \quad (3)$$

where the Ising spins  $\sigma_j$  take values  $\pm 1$  and the statistical weights of spin configurations are given by  $\exp(-H)$  (the temperature is incorporated in the parameters  $J$  and  $\Gamma$ ). This model is equivalent to the “weather model” considered in Ref. 3, and one finds two different FCS phases: “analytic” (A) and “nonanalytic” (NA). In the NA phase,  $\chi_0(\lambda)$  has a singularity at  $\lambda = \pi$ . The phase diagram is shown in Fig. 2a, in coordinates  $\Gamma$  and  $J$  [9, 10]. The phase-transition line is given by

$$\cosh \Gamma = e^{2J}. \quad (4)$$

This FCS phase transition has a simple physical inter-

pretation. If one considers the Jordan–Wigner string

$$V_\pi(j) = \prod_{k=j_0}^j \sigma_k \quad (5)$$

(with respect to some reference site  $j_0$ ), then the FCS phase transition corresponds to a nonanalyticity of the correlation length of the exponentially decaying correlation function  $\langle V_\pi(0)V_\pi(j) \rangle$ , as a function of the parameters  $J$  and  $\Gamma$ . In the A phase, the correlation function  $\langle V_\pi(0)V_\pi(j) \rangle$  exhibits a pure exponential (at  $\Gamma < 0$ ) or a staggered-exponential (at  $\Gamma > 0$ ) decay as a function of  $j$ , while in the NA phase there are additional *incommensurate oscillations* in  $j$ . Note that this “FCS phase transition” is not a phase transition in the usual sense, since the partition function of the Ising model does not have a singularity at the transition line.

4. *Spin-1/2 XY chain.*— We now turn to a quantum example where a nontrivial FCS phase diagram may be explicitly constructed: the spin-1/2 XY chain in a transverse magnetic field. The Hamiltonian of the system is [11]

$$\hat{H} = \sum_j \left( \frac{1+\gamma}{2} \sigma_j^x \sigma_{j+1}^x + \frac{1-\gamma}{2} \sigma_j^y \sigma_{j+1}^y - h \sigma_j^z \right). \quad (6)$$

Without loss of generality, we assume  $\gamma \geq 0$  and  $h \geq 0$ . We are interested in the FCS phase diagram with respect to the number of up spins (as the observable  $Q$  in our construction) at zero temperature (in the particular case of  $\gamma = 1$ , the FCS in this system was studied in Ref. 1). By the Jordan–Wigner transformation, this model can be mapped onto a quadratic fermionic system [11, 12], and the generating function  $\chi_\Sigma(\lambda)$  for a subchain of  $N$  sites can be written as a  $N \times N$  Toeplitz determinant. By a simple extension of the derivation in Ref. 7, we find

$$\chi_\Sigma(\lambda) = \det_{1 \leq j \leq k \leq N} \int_0^{2\pi} \frac{dq}{2\pi} \sigma(q, \lambda) e^{iq(j-k)} \quad (7)$$

with the *symbol*  $\sigma(q, \lambda)$  of the Toeplitz determinant given by [cf. Eq. (17) of the first paper of Ref. 7]

$$\sigma(q, \lambda) = \frac{1 + e^{i\lambda}}{2} + \left( \frac{1 - e^{i\lambda}}{2} \right) \frac{\cos q - h + i\gamma \sin q}{\sqrt{(\cos q - h)^2 + \gamma^2 \sin^2 q}}. \quad (8)$$

The exponential asymptotic dependence of such determinants on  $N$  is given by the Szegő formula [13]. It immediately produces the result for  $\chi_0(\lambda)$ :

$$\chi_0(\lambda) = \exp \int_0^{2\pi} \frac{dq}{2\pi} \ln \sigma(q, \lambda), \quad (9)$$

provided that  $\sigma(q, \lambda)$  has zero winding of the complex phase as  $q$  varies from 0 to  $2\pi$ . In the integral (9), the branch of the logarithm is chosen by the analytic continuation along the real axis of  $q$ , and the zero-winding

condition implies that  $\ln \sigma(2\pi, \lambda) = \ln \sigma(0, \lambda)$  under such an analytic continuation.

For some values of  $\gamma$ ,  $h$ , and  $\lambda$ , however, the symbol (8) may have winding number one [so that  $\ln \sigma(2\pi, \lambda) = \ln \sigma(0, \lambda) + 2\pi i$ ]. In this case, a modification of the Szegő formula applies [14]:

$$\chi_0(\lambda) = -\exp \left( \int_0^{2\pi} \frac{dq}{2\pi} \ln [\sigma(q, \lambda) e^{-iq}] + iq_0 \right), \quad (10)$$

where  $q_0$  is the location of the singularity of  $\sigma(q, \lambda)$  in the upper half plane of  $q$  with the smallest imaginary part.

A tedious, but straightforward application of Eqs. (9) and (10) allows us to calculate explicitly  $\chi_0(\lambda)$  at all values of  $\gamma$  and  $h$ . As a result, we find three FCS phases shown in Fig. 2b: two nonanalytic phases and an analytic one. Note that the same phase diagram appeared previously in the analysis of spin correlations [12] and of EFP [7]. The generating functions  $\chi_0(\lambda)$  at typical points in each of the three phases are shown in Fig. 3. Below we summarize some properties of these phases in terms of FCS.

*Nonanalytic I phase (NA I):*  $\gamma^2 + h^2 < 1$ . In this phase, we find (assuming  $\lambda \in [-\pi, \pi]$ ,  $\gamma \geq 0$ ,  $h \geq 0$ ) for the absolute value of  $\chi_0(\lambda)$ :

$$|\chi_0(\lambda)| = \sqrt{\frac{1 + \gamma \cos \lambda}{1 + \gamma}}. \quad (11)$$

Expressions for the phase of  $\chi_0(\lambda)$  following from Eqs. (9) and (10) are lengthy for all the three phases, and we do not present them here, except in several particular cases where they can be considerably simplified. In the NA I phase, the only singularity of  $\chi_0(\lambda)$  is a phase jump at  $\lambda = \pi$ :

$$\text{Im} \ln \chi_0(\pm\pi) = \pm \left( \pi - \arccos \frac{h}{\sqrt{1 - \gamma^2}} \right). \quad (12)$$

In several special cases,  $\chi_0(\lambda)$  takes a particularly simple form. At  $\gamma = 0$ ,

$$\chi_0(\lambda) = \exp \left[ i\lambda \left( 1 - \frac{1}{\pi} \arccos h \right) \right] \quad (13)$$

[in this case, the spin chain is equivalent to free fermions with the density  $1 - (1/\pi) \arccos h$ ].

At  $h = 0$ ,

$$\chi_0(\lambda) = e^{i\lambda/2} \sqrt{\frac{1 + \gamma \cos \lambda}{1 + \gamma}}. \quad (14)$$

At the phase boundary  $\gamma^2 + h^2 = 1$ ,

$$\chi_0(\lambda) = p e^{i\lambda} + 1 - p, \quad p = \frac{1}{2} \left( 1 + \sqrt{\frac{1 - \gamma}{1 + \gamma}} \right), \quad (15)$$

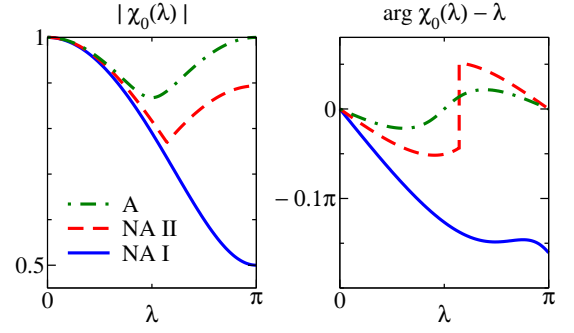


FIG. 3: The absolute value and the phase of the generating function  $\chi_0(\lambda)$  at typical points in each of the three FCS phases of the XY chain. For better visualization, the linear part  $\lambda$  is subtracted from the phase of  $\chi_0(\lambda)$ . In each panel, the three curves are (from bottom to top): NA I phase (solid line),  $\gamma = 0.6$ ,  $h = 0.7$ ; NA II phase (dashed line),  $\gamma = 0.5$ ,  $h = 0.95$ ; A phase (dash-dotted line),  $\gamma = 0.5$ ,  $h = 1.01$ .

which corresponds to independent spins with the probability  $p$  of pointing up [12].

Note that Eq. (11) is not even in  $\gamma$ , and therefore  $\chi_0(\lambda)$  depends nonanalytically on  $\gamma$  across the line  $\gamma = 0$ .

*Nonanalytic II phase (NA II):*  $\gamma^2 + h^2 > 1$ ,  $|h| < 1$ . In this phase (again assuming  $\lambda \in [-\pi, \pi]$ ,  $\gamma \geq 0$ ,  $h \geq 0$ ),  $\chi_0(\lambda)$  has phase jumps at points  $\pm\lambda_c$  given by

$$\cos \lambda_c = -\frac{1 - z_1^2}{\gamma(1 + z_1^2)}, \quad z_1 = \frac{h + \sqrt{\gamma^2 + h^2 - 1}}{1 + \gamma}. \quad (16)$$

For the absolute value of  $\chi_0(\lambda)$ , we find

$$|\chi_0(\lambda)| = \begin{cases} \sqrt{\frac{1 + \gamma \cos \lambda}{1 + \gamma}} & |\lambda| < \lambda_c, \\ z_1 \sqrt{\frac{1 - \gamma \cos \lambda}{1 + \gamma}} & |\lambda| > \lambda_c. \end{cases} \quad (17)$$

Once again, we do not present here full expressions for the phase of  $\chi_0(\lambda)$ . The phase jump at  $\lambda_c$  is given by

$$\text{Im} \ln \frac{\chi_0(\lambda_c + 0)}{\chi_0(\lambda_c - 0)} = \arccos \frac{h(1 + z_1^2)}{2z_1} \quad (18)$$

(this jump tends to zero at the phase boundary). At the boundary with the NA I phase,  $\lambda_c \rightarrow \pi$ , and  $\chi_0(\lambda)$  is given by Eq. (15). At the boundary with the Analytic phase  $h = 1$ , one finds  $\lambda_c \rightarrow \pi/2$ . In the whole NA II phase,  $\text{Im} \ln \chi_0(\pm\pi) = \pm\pi$ , which implies that  $\chi_0(\lambda)$  is smooth at  $\lambda = \pi$ .

*Analytic phase (A):*  $|h| > 1$ . In this phase,  $\chi_0(\lambda)$  has no singularities in  $\lambda$ . For its absolute value we find:

$$|\chi_0(\lambda)| = \sqrt{\frac{h + \sqrt{\gamma^2 \cos^2 \lambda + h^2 - 1}}{h + \sqrt{\gamma^2 + h^2 - 1}}}. \quad (19)$$

Throughout this phase,  $\text{Im} \ln \chi_0(\pm\pi) = \pm\pi$  (just like in the NA II phase). At  $\gamma = 0$ , the generating function takes

the particularly simple form  $\chi_0(\lambda) = i\lambda$  [a completely filled band of free fermions].

We notice a remarkable property of the A phase: throughout this phase,  $\chi_0(\pi) = -1$ . This means that the correlations of the Jordan–Wigner operators (5) [where  $\sigma_k$  denote now the  $z$  components  $\sigma_k^z$ ] decay slower than exponentially. This property was described in Ref. 1 as “confinement of dual domain walls”, and it can also be understood in the fermionic language. At  $|h| > 1$ , fermions form a completely filled (or completely empty) band, and the anisotropy terms (involving  $\gamma$ ) introduce local Cooper pairs, which, however, change the number of particles by two. Thus the expectation value of the Jordan–Wigner parity string  $\langle V_\pi(0)V_\pi(j) \rangle$  is only affected by Cooper pairs intersecting one of the end points of the string (0 or  $j$ ). But, since pairs are local, the number of such pairs remains finite for long strings, which leads to a saturation of the correlations  $\langle V_\pi(0)V_\pi(j) \rangle$  at large  $j$ .

The FCS phase diagram of the XY chain (Fig. 2b) coincides with that obtained from spin correlations [12], since, by the Jordan–Wigner transformation, transverse spin operators  $\sigma_j^\pm$  are represented by the product of the string operator (5) and a fermion operator [11]. As a consequence, transverse spin correlations are given by the Toeplitz determinants which differ from Eqs. (7) and (8) only by a shift of the winding number (and by fixing  $\lambda = \pi$ ) [12]. Therefore, spin correlations produce the same phase diagram, where phases differ from each other by the presence or absence of incommensurate oscillations in  $\langle \sigma_j^+ \sigma_0^- \rangle$  and by pre-exponential factors in their  $j$  dependence [12] (the latter is disregarded in our present work [5]).

The phase diagram in Fig. 2b also resembles those based on EFP in Ref. 7 and on entanglement entropy in Ref. 15. Indeed, the EFP is given by the same Toeplitz determinant (7), (8) with  $\lambda \rightarrow i\infty$  [7], and the entanglement entropy depends on the spectrum of a closely related block Toeplitz matrix [15]. Therefore the phase boundaries which are determined by the geometry of the square-root branching points in Eq. (8) coincide in all the three problems. However the FCS classification contains additional details related to the positions of  $\lambda$ -dependent logarithmic branching points in Eq. (9).

Finally, we remark that, even though the XY spin chain considered in our work maps onto a quadratic fermionic system, it does not obey the theorem on factorization of FCS for noninteracting fermions of Ref. 16: the phase NA II with a singularity at an intermediate value of  $\lambda$  would not be allowed by that theorem. The reason for this discrepancy is that the corresponding fermionic system contains pairing terms [11], and the theorem of Ref. 16 is not, in general, valid for quadratic Hamiltonians with pairing (see also discussion in the supplementary material of the last paper of Ref. 8).

**5. Conclusion.**— We have proposed a classification scheme of thermodynamic phases in terms of the analytical properties of the extensive part of FCS for a suitably chosen discrete observable. This proposal was illustrated with two examples where FCS could be calculated analytically. In more complicated systems, it may be possible to access FCS numerically.

In the present paper, we focused on analytic properties of the generating function  $\chi_0(\lambda)$  at real values of  $\lambda$ . It may also be instructive to analyze, more generally, the structure of singularities of  $\chi_0(\lambda)$  in the complex plane of  $e^{i\lambda}$ . This would provide a connection to the theory of EFP [6, 7] and to the analysis of FCS for noninteracting fermions in Ref. 16.

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